Lecture 21

Monday, 14 November 2022 1:50 PM

Multi - Parameter Mech. Design.

Setting:

- n bidders,
- set Ω of outcomes
- bidder i has private value $v_i(\omega)$ $\forall \omega \in \Omega$
  (prior-free, worst-case setting)

objective: DSIC mechanisms to maximize welfare
$M = (x, p)$

Earlier, in the single-parameter setting, sufficient for $x_i(b_i)$ to be monotone in b_i
In the multi parameter setting, not clear what this means...

Still, since our goal is SW maximization, if we assume truthfulness, then our allocation rule is:

$$\text{Choose } x(b) \in \arg\max_{\omega \in \Omega} \sum_i b_i(\omega)$$

Now for truthful payments:

Aside: let's consider the single item auction again.
we assigned to highest bidder, charged the 2nd highest bid. This was unique DSIC payment, assuming $x_i(b) = 0 \Rightarrow p_i(b) = 0$.
If we relax this, consider the following mechanism:

- assign item to highest bidder
  say i^* is the highest bidder, w/ bid b_{i^*}
- Pay all other agents b_{i^*} (for $j \neq i$, $p_j(b) = -b_{i^*}$)
  can check this is DSIC

Note that the payment to agent j:

1. is independent of j's bid (but does depend on the allocation x)
2. "aligns incentives": assuming truthful bids, each bidder's utility is the SW, which is also what the mechanism designer is looking to maximize.

More generally, in the multi-parameter setting:

- choose $\omega^* \in \Omega$ to maximize $\sum_i b_i(\omega^*)$
- $\forall i$, $p_i(b) = -\sum_{j \neq i} b_j(\omega^*)$

hence, $\forall i$,
$$u_i(b) = v_i(x(b)) - p_i(b)$$
$$= v_i(x(b)) + \sum_{j \neq i} b_j(\omega^*) = v_i(\omega^*) + \sum_{j \neq i} b_j(\omega^*)$$

clearly, each agent maximizes her revenue by bidding $b_i = v_i$.

The problem is that payments are -ve.
To make payments +ve, we add a term:

$$p_i(b) = -\sum_{j \neq i} b_j(\omega^*) + \max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega)$$

note that $p_i(b) \geq 0$, $\forall i$.

Theorem: The mechanism $M = (x, p)$ with

$$x(b) \in \arg\min_{\omega \in \Omega} \sum_i b_i(\omega)$$

$$\& \quad p_i(b) = \left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega)\right) - \sum_{j \neq i} b_j(x(b))$$

is DSIC, SW-maximizing, & has nonnegative payments for all agents.

(proof is trivial)

Note that that these payments can be interpreted in a few different ways:

1. $$p_i(b) = \underbrace{\left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega)\right)}_{\text{SW of other agents if } i \text{ was not present}} - \underbrace{\sum_{j \neq i} b_j(x(b))}_{\text{SW of other agents due to } i\text{'s presence}}$$

   i has to the harm she caused to the SW of the other agents.

2. $$p_i(b) = b_i(x(b)) - \left[\sum_j b_j(x(b)) - \max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega)\right]$$

   i has to pay her bid, but gets a discount equal to the increase in SW due to her presence.

However, there are multiple issues with these mechanisms:

1. Ω is typically v. large. Eg., consider m items, n bidders.
   # possible allocations is $(n+1)^m$.
   How do bidders even communicate b_i?
2. computational issues:
   given b_i's in some compact form, if $|\Omega|$ is large, how do you compute $\max_{\omega \in \Omega} \sum_i b_i(\omega)$?
3. secondary IC considerations:
   2 items A & B
   a. 2 bidders: $v_1(A, B) = 1$, $v_1(A) = v_1(B) = v_1(\phi) = 0$
      $v_2(A, B) = v_2(A) = 1$, $v_2(B) = v_2(\phi) = 0$
      - whoever wins, pays 1.

   b. 3 bidders, $v_3(A, B) = v_3(B) = 1$, $v_3(A) = v_3(\phi) = 0$
      - 2 & 3 win, pay 0!

   (thus, agent 2 has an incentive to create "spurious" agents...)